Probability and Random Processes

ECS 315

Asst. Prof. Dr. Prapun Suksompong

prapun@siit.tu.ac.th

3 Classical Probability

Office Hours:

BKD, 6th floor of Sirindhralai building

Tuesday	**9:00-10:00**
Wednesday	**14:20-15:20**
Thursday	**9:00-10:00**

Real coins are biased

- From a group of Stanford researchers

DYNAMICAL BIAS IN THE COIN TOSS

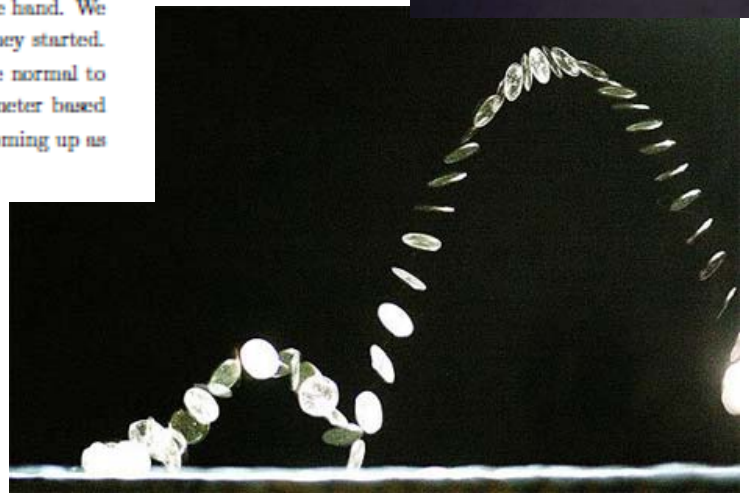
Persi Diaconis
Departments of Mathematics and Statistics
Stanford University

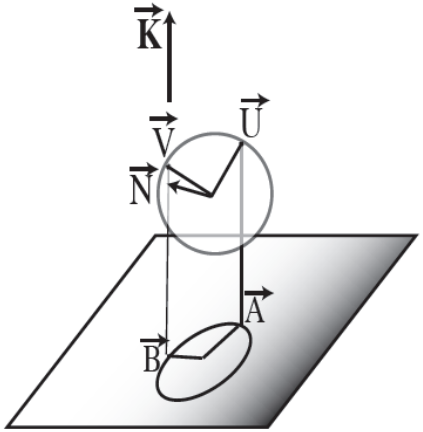
Susan Holmes
Department of Statistics
Sequoia Hall
Stanford University

Richard Montgomery
Department of Mathematics
University of California
Santa Cruz

Abstract

We analyze the natural process of flipping a coin which is caught in the hand. We prove that vigorously-flipped coins are biased to come up the same way they started. The amount of bias depends on a single parameter, the angle between the normal to the coin and the angular momentum vector. Measurements of this parameter based on high-speed photography are reported. For natural flips, the chance of coming up as started is about .51.

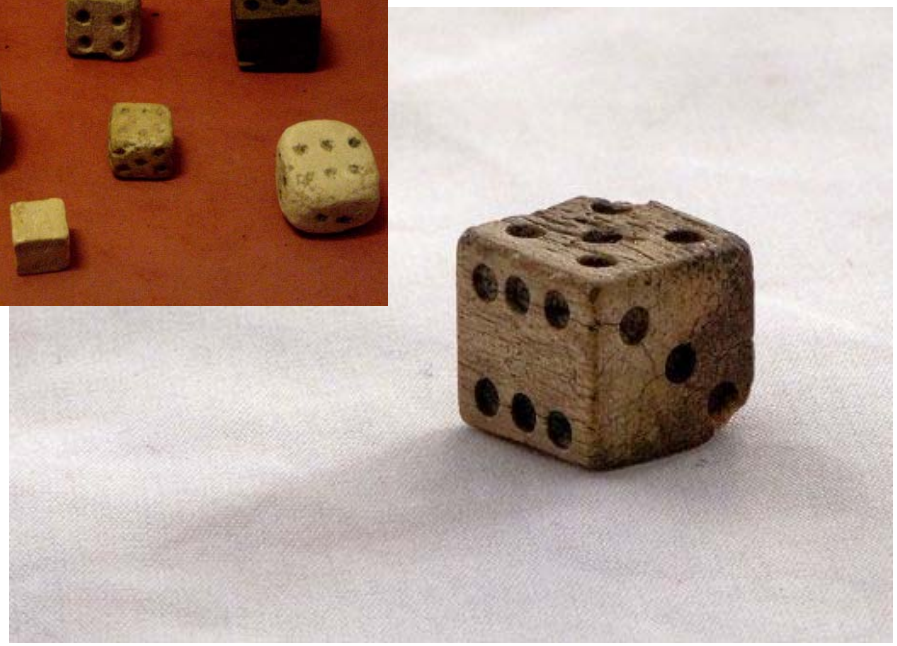




http://gajitz.com/up-in-the-air-coin-tosses-not-as-neutral-as-you-think/
http://www.codingthewheel.com/archives/the-coin-flip-a-fundamentally-unfair-proposition
http://www-stat.stanford.edu/~susan/papers/headswithJ.pdf

The word “dice”

- Historically, **dice** is the plural of **die**.
- In modern standard English, **dice** is used as both the singular and the plural.



Example of 19th Century bone dice

Gaming Dice

- Cheaply made.
- Have rounded edges
- **Pips**
  - Indentations on the side of dice (the little dots cut out from each side) to indicate the value of the face
  - This produces an uneven balance as the "six" side has more pips (less material/weight) than the "one" side.
- **1s are by far the most common result**
  - An experiment using Chessex and GW dice got 29% one on average.
- The amount of plastic saved from rounding the corners and hollowing out the pips of 2 dice actually gave them enough left over plastic to make a 3rd dice.

[http://www.dakkadakka.com/wiki/en/That%27s_How_I_Roll_-_A_Scientific_Analysis_of_Dice]

AIP | Chaos
An Interdisciplinary Journal of Nonlinear Science
25TH ANNIVERSARY

The three-dimensional dynamics of the die throw

M. Kapitaniak[1,2], J. Strzalko[1], J. Grabski[1] and T. Kapitaniak[1]

+ VIEW AFFILIATIONS

Chaos **22**, 047504 (2012); http://dx.doi.org/10.1063/1.4746038

A three-dimensional model of a die throw which considers the die bounces with dissipation on the fixed and oscillating table has been formulated. It allows simulations of the trajectories for dice with different shapes. Numerical results have been compared with the experimental observation using high speed camera. It is shown that for the realistic values of the initial energy the probabilities of the die landing on the face which is the lowest one at the beginning is larger than the probabilities of landing on any other face. We argue that non-smoothness of the system plays a key role in the occurrence of dynamical uncertainties and gives the explanation why for practically small uncertainties in the initial conditions a mechanical randomizer approximates the random process.

Key Topics

Friction
7.0

Chaos
6.0

Cameras
4.0

Chaotic dynamics
4.0

Classical mechanics
4.0

Received 16 February 2012 **Accepted** 31 July 2012 **Published online** 14 December 2012

Non-Symmetry in Dice

ลูกเต๋าพบในเมือง
โบราณอู่ทอง
จังหวัดสุพรรณบุรี
จัดแสดงใน
พิพิธภัณฑสถาน
แห่งชาติ อู่ทอง

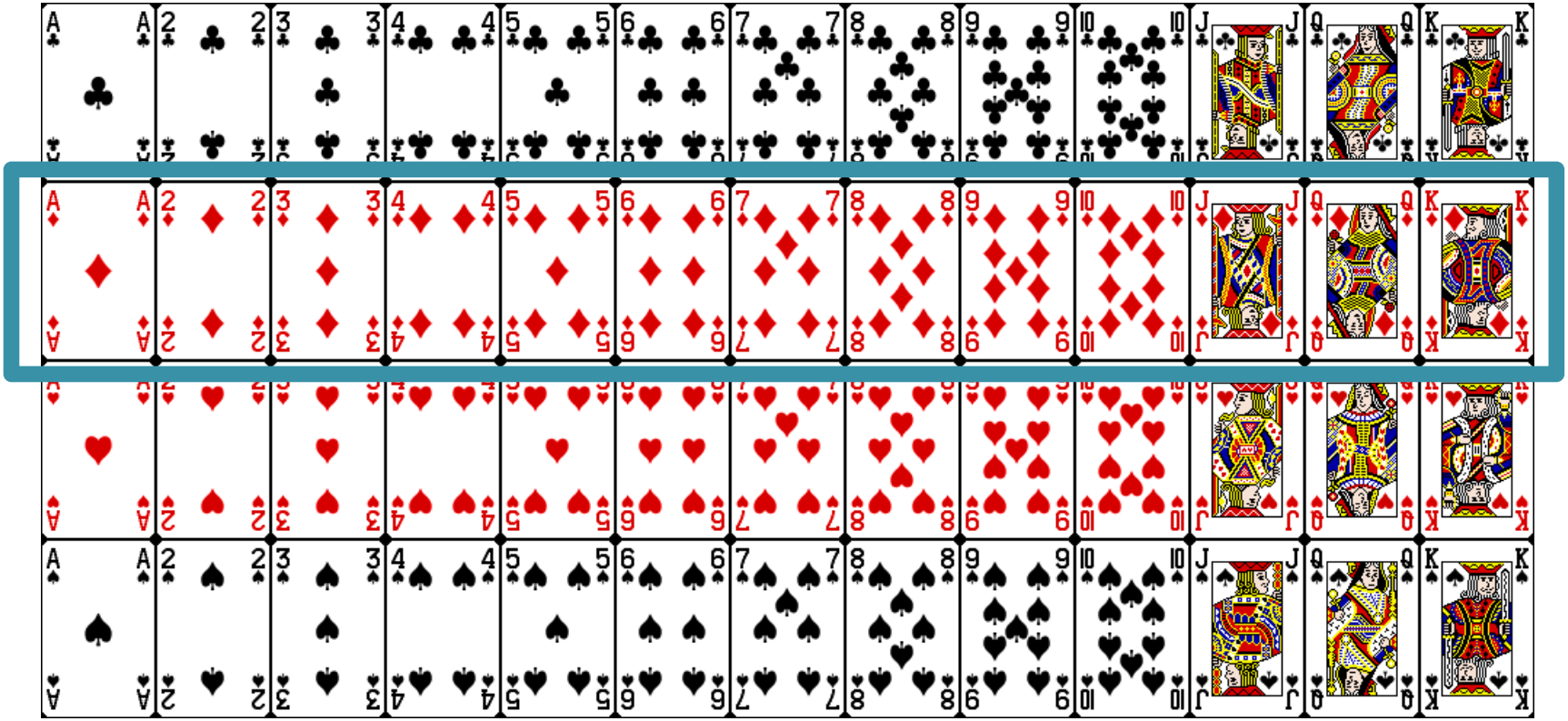
[http://www.museum-press.com/content--4-5600-111840-1.html]

Loaded Dice

- A loaded, weighted or crooked die is one that has been tampered with so that it will land with a specific side facing upwards more or less often than a fair die would.

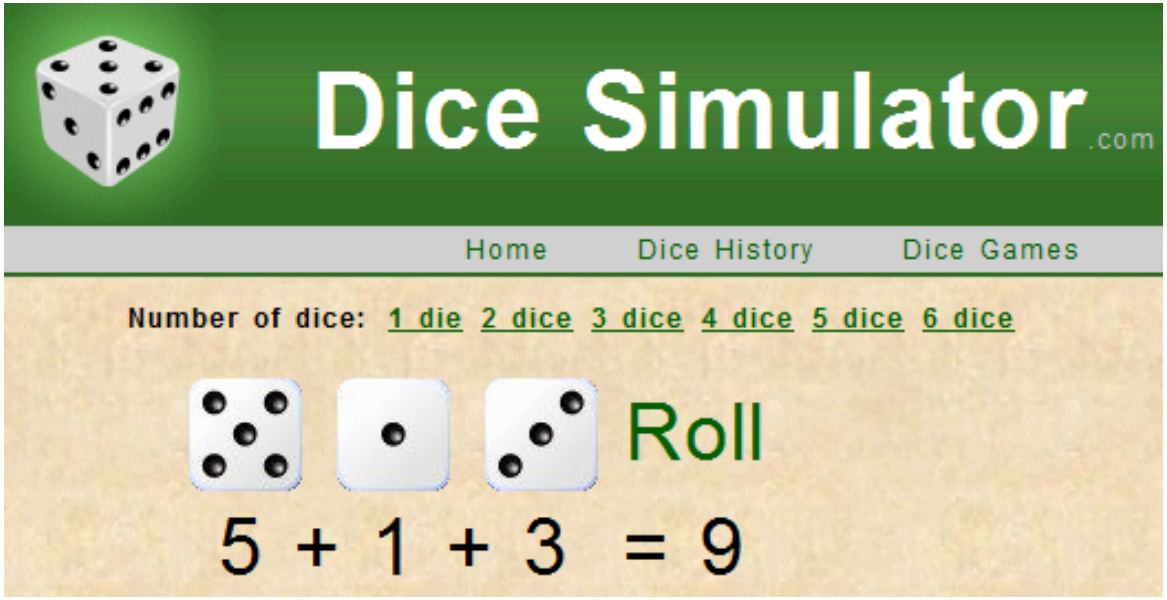
Example

- In drawing a card from a deck, there are 52 equally likely outcomes, 13 of which are **diamonds**. This leads to a probability of 13/52 or 1/4.

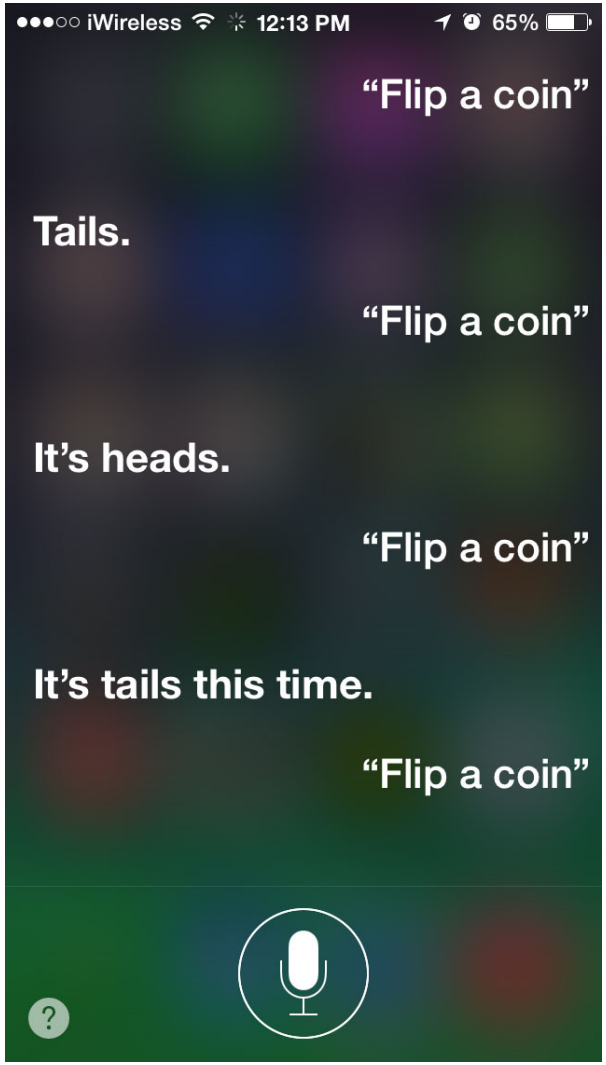


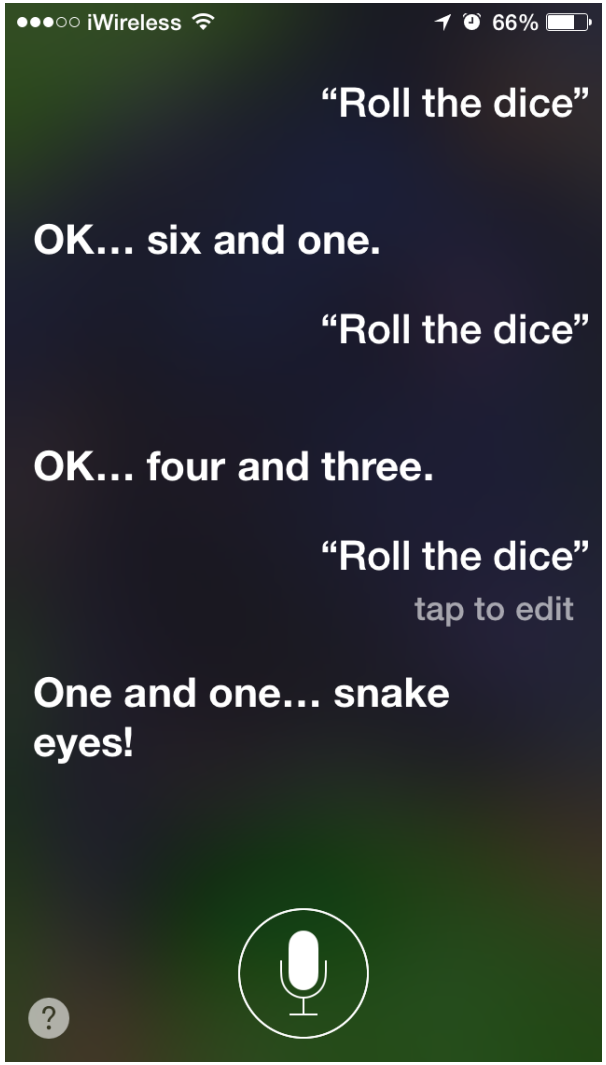
Dice Simulator

- http://www.dicesimulator.com/
- Support up to 6 dice and also has some background information on dice and random numbers.



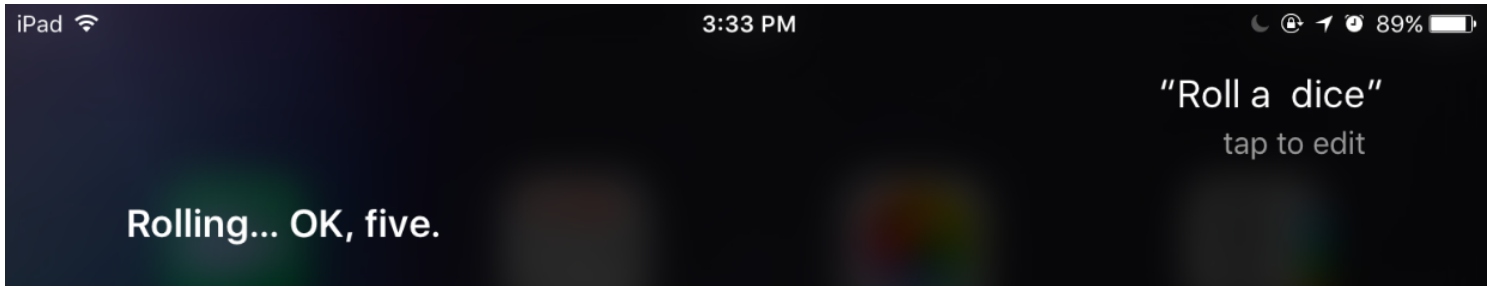
Roll the dice or flip a coin with Siri

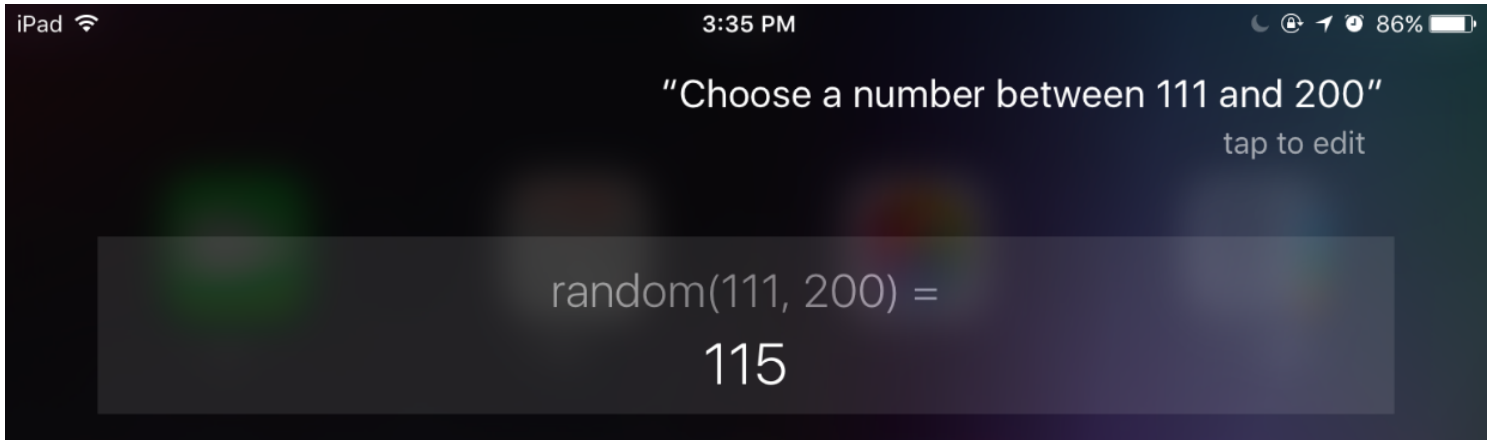




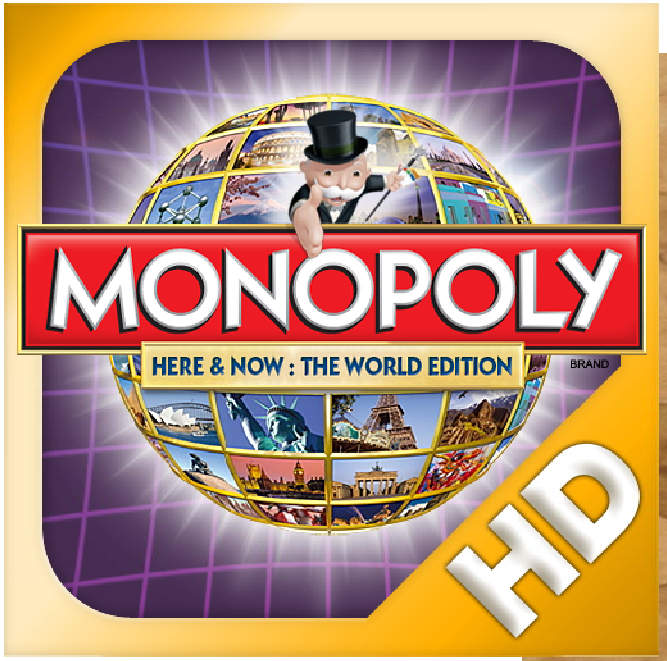
[http://www.iphonelife.com/blog/31961/tip-day-siri-can-flip-coin-roll-dice-and-make-hard-parenting-choices-you]

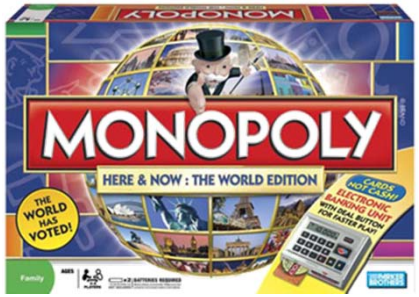
Do more with Siri





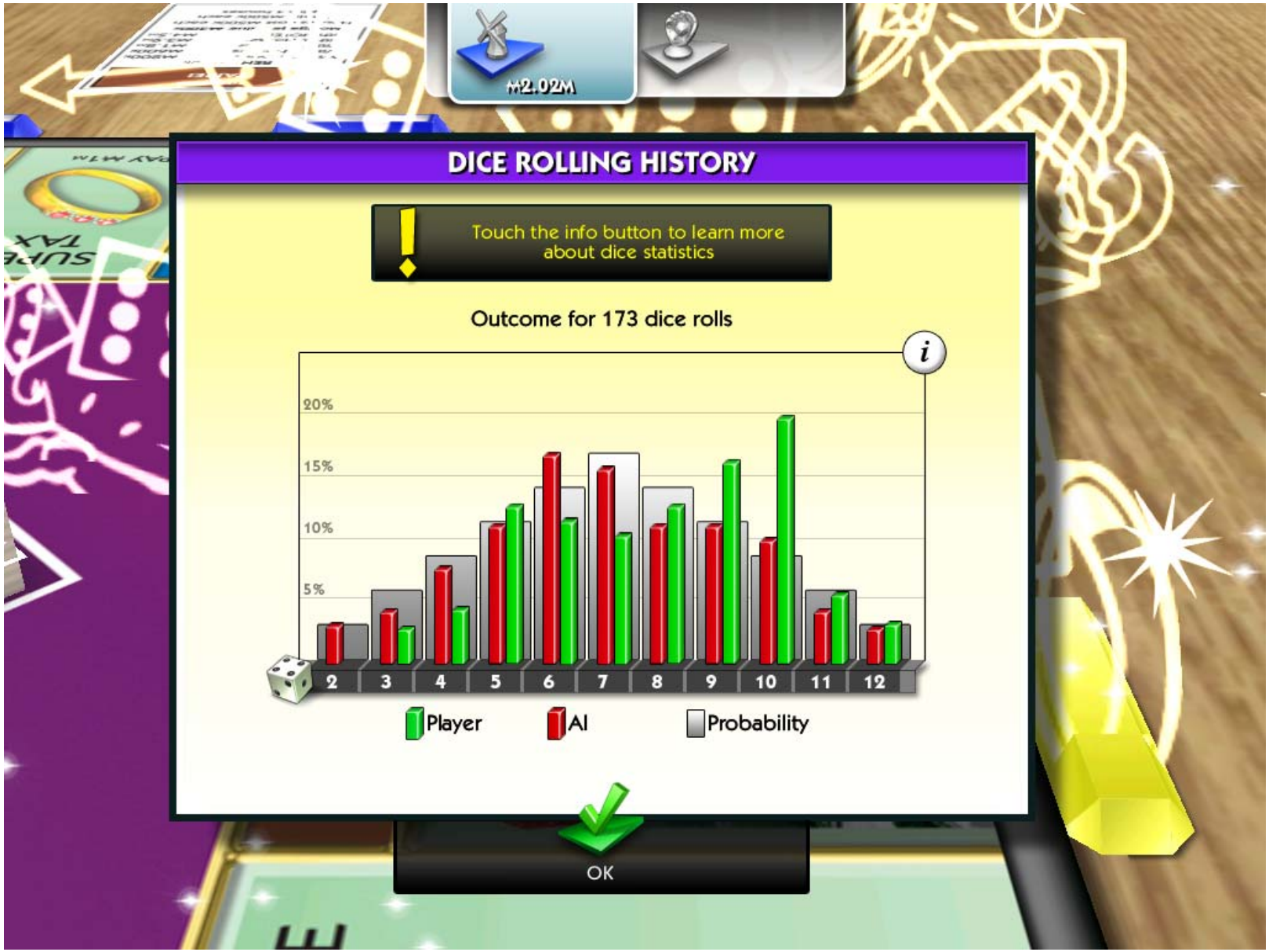
Two Dice


MONOPOLY
HERE & NOW : THE WORLD EDITION
BRAND
HD


MONOPOLY
HERE & NOW : THE WORLD EDITION
THE WORLD HAS VOTED!
CARDS NOT CASH!
ELECTRONIC BANKING UNIT WITH REAL MOTION FOR FASTER PLAY!

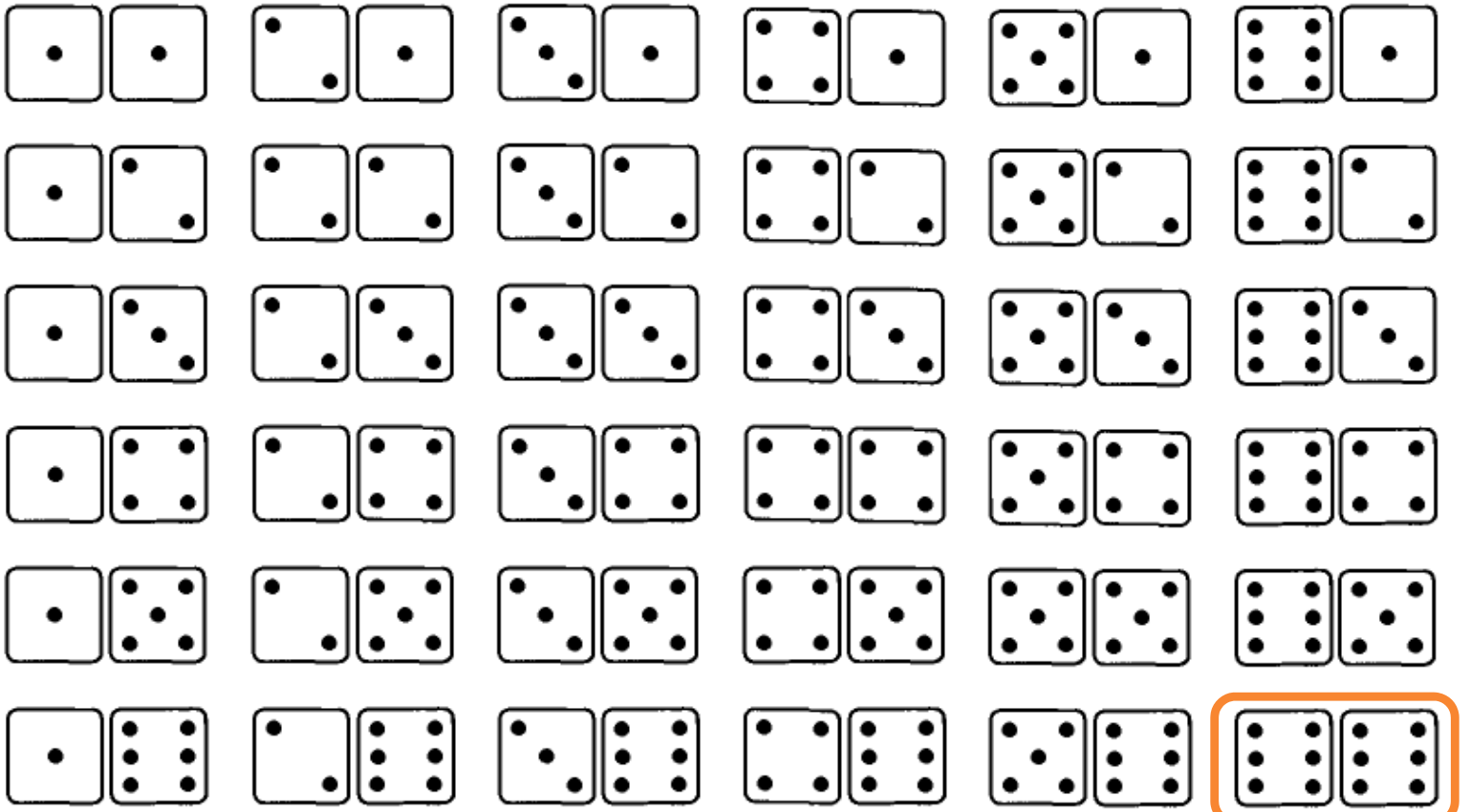

M3.65M
M15.32M
M9.52M
M11.86M
MONOPOLY
HERE & NOW : THE WORLD EDITION
JAIL
VISITING
ATHENS
BARCELONA
CHANCE
TOKYO
INCOME TAX
TAIPEI
COMMUNITY CHEST
GDYNIA
SUPER TAX
MONTREAL
COLLECT M2M SALARY AS YOU PASS
GO

Two-Dice Statistics

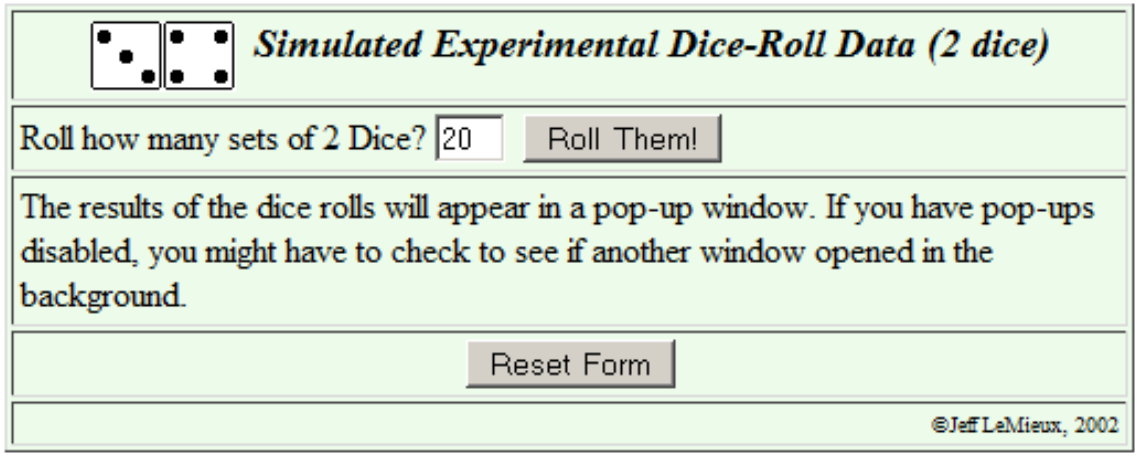


Two Dice

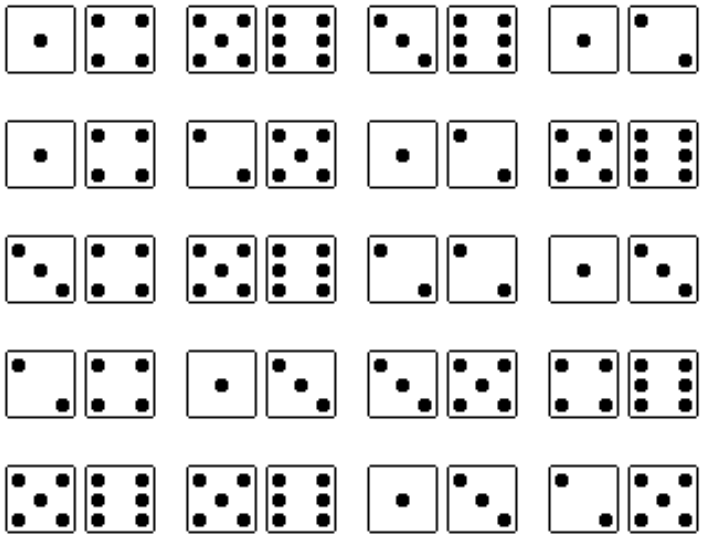
- A pair of dice



Two dice: Simulation

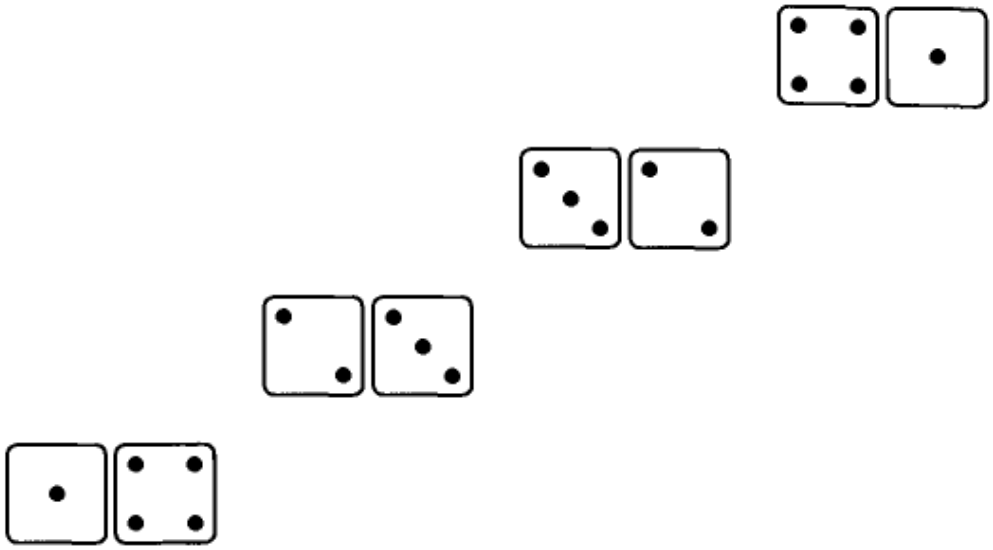


[http://www2.whidbey.net/ohmsmath/webwork/javascript/dice2rol.htm]



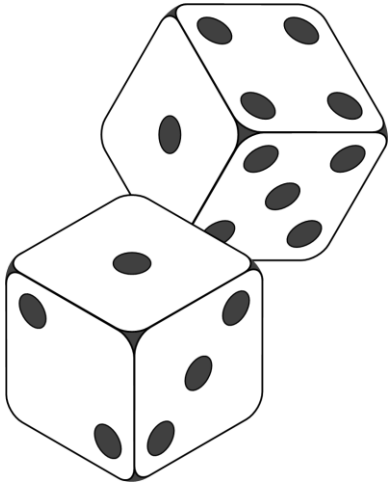
Two dice

- Assume that the two dice are fair and independent.
- P[sum of the two dice = 5] = 4/36



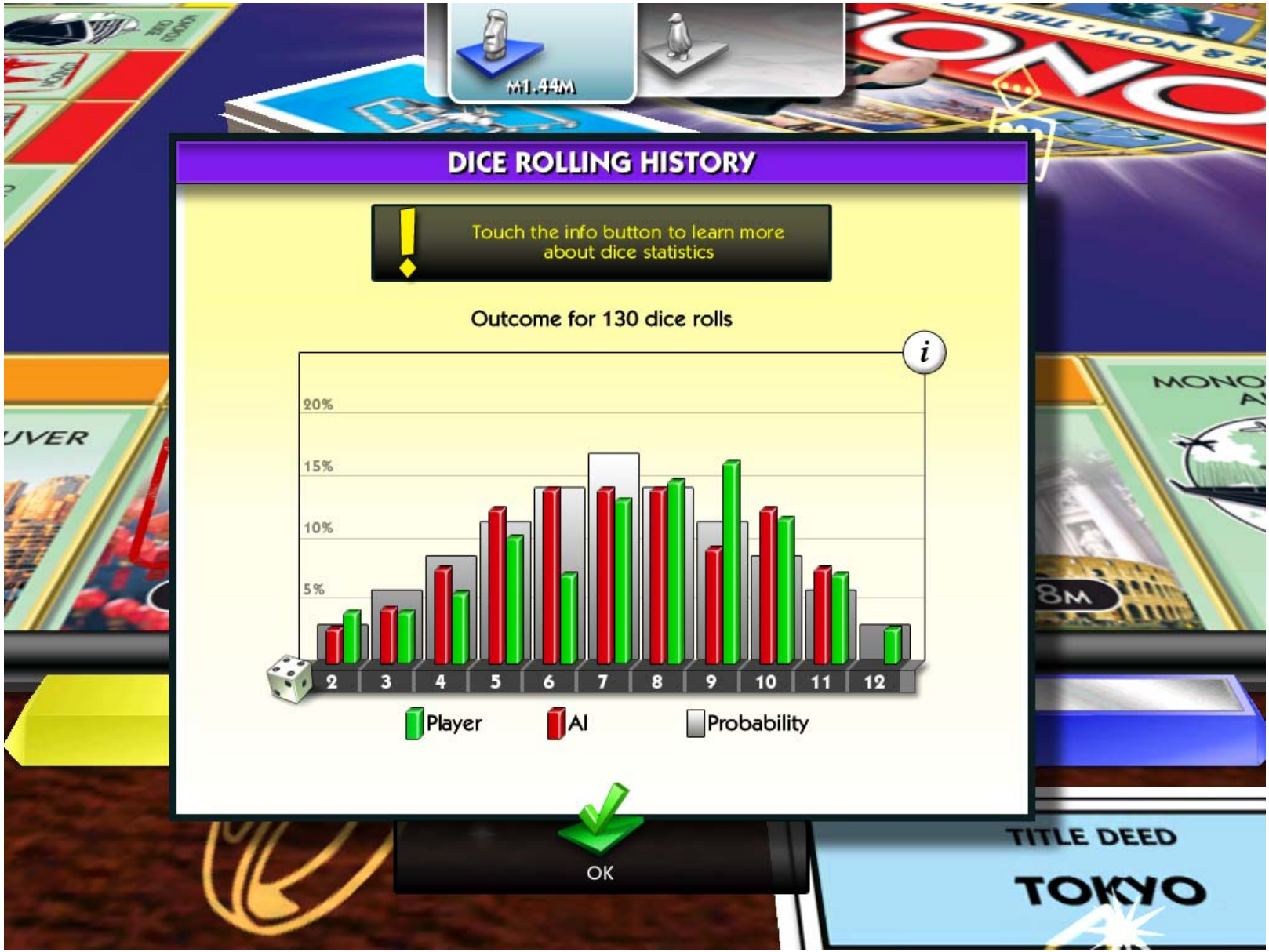
Two dice

- Assume that the two dice are fair and independent.



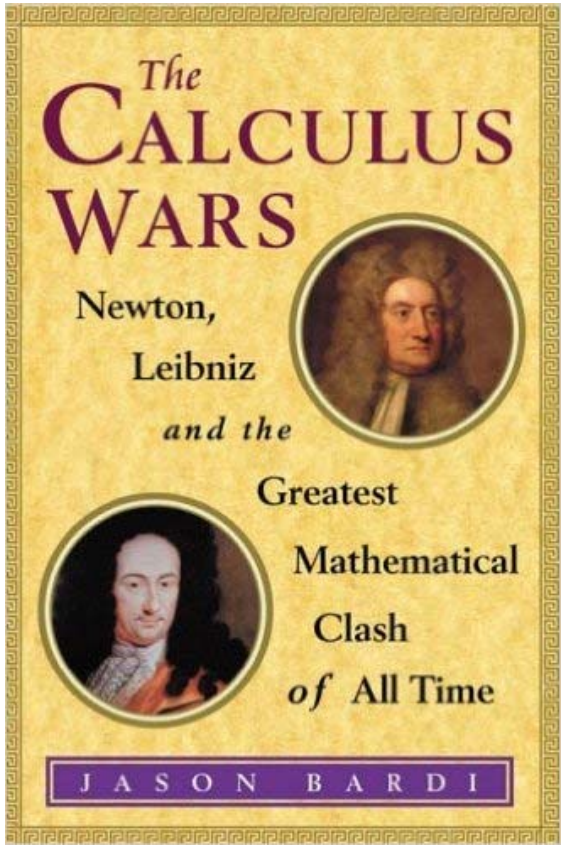
DICE CHART		
ROLL		PROBABILITY →
2		1/36
3		2/36
4		3/36
5		4/36
6		5/36
7		6/36
8		5/36
9		4/36
10		3/36
11		2/36
12		1/36

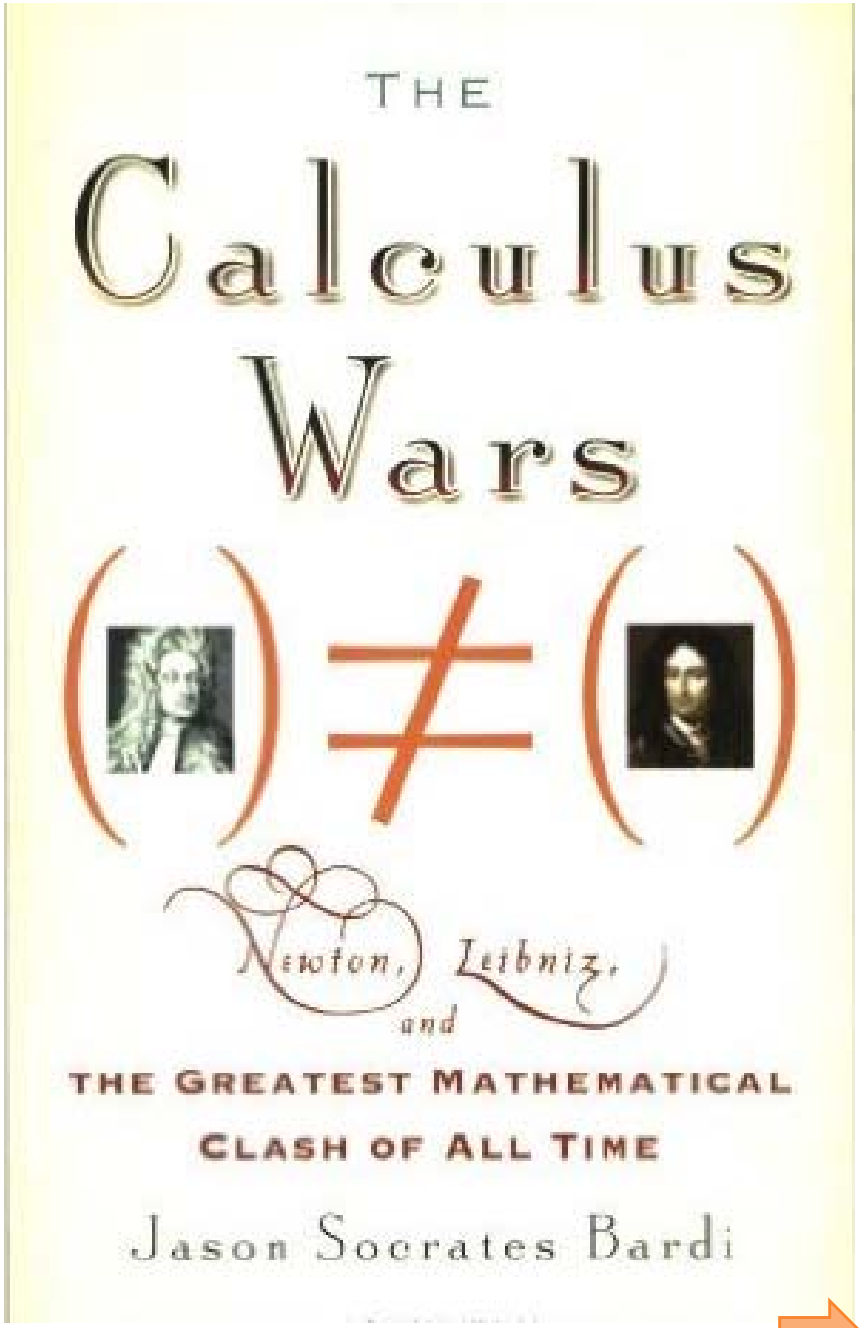
Two-Dice Statistics



Calculus War

- Nontechnical account of the battle between Newton and Leibniz over **who invented calculus**.





Calculus War

Calculus War: Leibniz